

Make an Identity

[Also posted at <https://eugenebo.wordpress.com/2017/02/15/make-an-identity/> and <https://eugenebo.livejournal.com/205432.html>]

You probably have seen mathematical puzzles of this type before. Someone gives you several numbers and asks to place arithmetic and “other mathematical signs” between or around them so that the resulting expression is evaluated as some target number.

For example, if the numbers are **2, 3, 4**, with the target is 10, one of the solutions might be:

$$10 = -(2-3*4)$$

Or here is another:

$$10 = 2*3 + 4$$

Here, I would like to present a general solution to this problem when mathematical signs include elementary analytic functions and all numbers are integers.

Let's start with the core relation between hyperbolic functions (https://en.wikipedia.org/wiki/Hyperbolic_function):

$$\cosh^2(x) - \sinh^2(x) = 1$$

From that, the relation between the inverse hyperbolic functions immediately follows:

$$\cosh(\operatorname{Arcsinh}(x)) = \sqrt{1+x^2}$$

Using $x = \sqrt{k}$, one arrives then to a relation that converts k into $k+1$:

$$\cosh(\operatorname{Arcsinh}(\sqrt{k})) = \sqrt{1+k} \quad (1)$$

By applying that formula repeatedly as many times as needed, we can increment integers. For example:

$$\cosh(\operatorname{Arcsinh}(\cosh(\operatorname{Arcsinh}(\cosh(\operatorname{Arcsinh}(\cosh(\operatorname{Arcsinh}(\cosh(\operatorname{Arcsinh}(2)))))))))) = 3$$

(you can check that with a calculator)

From now on, the rest is easy. We just apply that formula to one of the given numbers, and play it as many times as needed to produce **10**, or whatever the target is.

What do you do with other numbers? You decrement them away to zeros with a reverse of formula (1):

$$\sinh(\operatorname{Arccosh}(\sqrt{k})) = \sqrt{k-1} \quad (2)$$

So, for instance, if you wanted to make **10** out of numbers **8, 3, 5, 2**, you would've written:

$$10 = \text{Cosh}(\text{Arcsinh}(\dots(\text{Cosh}(\text{Arcsinh}(8)))\dots)) + \text{Sinh}(\text{Arccosh}(\dots(\text{Sinh}(\text{Arccosh}(3)))\dots)) + \text{Sinh}(\text{Arccosh}(\dots(\text{Sinh}(\text{Arccosh}(5)))\dots)) + \text{Sinh}(\text{Arccosh}(\dots(\text{Sinh}(\text{Arccosh}(2)))\dots))$$

Where the first pair of calls is repeated **36** times, the second **9** times, the third **25** times, and the last one **4** times.

Obviously, further optimizations are possible, but this is sufficient as a solution.

Have a nice day,
Eugene

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